Student ID (not your netID):_________________

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___________________________________
(Signature)

Instructions
1. 2 hour exam, total number of points = 25.
2. Numeric calculators and one 8.5x11” sheet (both sides) of notes are permitted.
3. Write in the space provided. You may write on the back of a page for extra space.
4. You may not ask any questions. If you are unclear on a question, write down your interpretation and state any assumptions in your answer.
5. There is no partial credit for multiple-choice questions. For other questions, show your work when appropriate: correct work and correct reasoning may receive partial credit.

Grading

<table>
<thead>
<tr>
<th>Questions</th>
<th>Possible Points</th>
<th>Deductions</th>
<th>Total Points</th>
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<tr>
<td>1 – 2</td>
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<td>3 – 4</td>
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</table>
The following description applies to questions 1-2.

Gaur’s Tire Shop sells tires and rims at a fairly steady rate of 85 and 25 per week. A typical tire costs $180 and a typical rim costs $650. Gaur’s uses an annual holding cost rate of 15%. Assume that there are 50 weeks in a year.

1. [2 points] Gaur’s orders its tires from Tires-R-Us which charges a fixed cost of $750 per order, and its rims from Rims-R-Us which charges a fixed cost of $2100 per order. What is the order size that Gaur’s should use if it wishes to minimize its annual holding and ordering cost associated with its tires (round up any fractional units)?

\[
K = 750; \quad D = 50 \times 85 = 4250 \text{ per year}; \quad h = 0.15 \times 180 = 27/\text{unit-year}
\]

\[
Q^* = \sqrt{\frac{2 \times K \times D}{h}} = \sqrt{\frac{2 \times 750 \times 4250}{27}} = 485.91 = 486
\]

Note: Most students got this correct.

2. [2 points] Tires-R-Us and Rims-R-Us merged and Gaur’s now has the ability to combine orders for tires and rims at a fixed cost of $2500 per order (i.e. tires and rims can be shipped together). Gaur’s has decided to place an order every 12 weeks. What is the resulting annual total cost under this new policy (holding and ordering cost)?

If an order is placed every 12 weeks, then:

Orders per year = \( \frac{50}{12} = 4.167 \)

Order quantity for tires = \( 85 \times 12 = 1020 \)

Order quantity for rims = \( 25 \times 12 = 300 \)

Holding cost per tire = \( 0.15 \times 180 = 27/\text{unit-year} \)

Holding cost per rim = \( 0.15 \times 650 = 97.5 \)

Annual Cost = \( 2500 \times \left( \frac{50}{12} \right) + \left( \frac{1}{2} \right) \times 1020 \times 27 + \left( \frac{1}{2} \right) \times 300 \times 97.5 = 38,812 \)

Note: Quite a few students correctly answered this more challenging question. It does not require one to apply the EOQ formula. Instead, one has to recognize that for annual costs, the shipments for two products can be combined, but holding costs must still be incurred for both products. Therefore, one must augment the annual cost equation to incorporate the two costs for holding product (and also consider the order quantity for tires and rims, assuming an order is placed every 12 weeks). We performed similar steps to this in class when we first derived the cost function.
The following description is for questions 3-4.

Nelson Corp. is considering investing in four bonds; a total amount of $1 million is available for investment. The expected annual return on each bond, the worst-case annual return, and the “duration” of each bond are given in the table below. (The expected and worst-case returns were estimated by economists at Nelson Corp. The duration is a measure of the bond’s sensitivity to interest rates.)

<table>
<thead>
<tr>
<th></th>
<th>Expected return</th>
<th>Worst case return</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond 1</td>
<td>13%</td>
<td>7%</td>
<td>3</td>
</tr>
<tr>
<td>Bond 2</td>
<td>8%</td>
<td>2%</td>
<td>4</td>
</tr>
<tr>
<td>Bond 3</td>
<td>12%</td>
<td>10%</td>
<td>7</td>
</tr>
<tr>
<td>Bond 4</td>
<td>14%</td>
<td>9%</td>
<td>9</td>
</tr>
</tbody>
</table>

Nelson wants to maximize the expected return from its bond investments, subject to two constraints:

- The worst case return of the bond portfolio must be at least 8%.
- The average duration of the portfolio must be no more than 6. For example, a portfolio that invests $600,000 in bond 1 and $400,000 in bond 4 has an average duration of

\[
\frac{600,000 \times 3 + 400,000 \times 9}{600,000 + 400,000} = 5.4
\]

Nelson wants to solve this problem using linear programming. Assume the decision variables B1, B2, B3, B4 are defined as $ investment in Bond 1, Bond 2, Bond 3, and Bond 4. No short sales are allowed. Do not assume that all $1 million must be invested. Answer the following questions by writing the appropriate algebraic mathematical formula, equation, or inequality using the variables defined above.

3. [1 point] Write the objective function for this linear program in terms of the variables B1, B2, B3 and B4, and whether it should be maximized or minimized.

Maximize Exp Return = 0.13*B1 + 0.08*B2 + 0.12*B3 + 0.14*B4

Note: You cannot divide by (B1+B2+B3+B4), this would be a non-linear objective function.

4. [2 points] Write the constraint for this linear program for the average duration of the portfolio to be no more than 6.

\[
(3 \times B1 + 4 \times B2 + 7 \times B3 + 9 \times B4) \leq 6 \times (B1 + B2 + B3 + B4)
\]

Or,

\[-3 \times B1 - 2 \times B2 + 3 \times B3 + 3 \times B4 \leq 0\]

Note: The constraint should be linearized. In other words, this is not acceptable:

\[
\frac{3 \times B1 + 4 \times B2 + 7 \times B3 + 9 \times B4}{B1 + B2 + B3 + B4} \leq 6
\]

Also, the problem says that one should not assume all $1 million is necessarily invested, so one cannot divide by $1,000,000.
The following description is for question 5.

Schmidt’s manufacturing firm uses control charts to monitor its process for making tablet cases. They have subgroup size of n=4, overall mean $\bar{X} = 29.9$, and average range $R = 7.3$. Using these data, the control limits are as follows:

- $\bar{X}$ UCL = 35.2,
- $\bar{X}$ LCL = 24.6,
- $R$ UCL = 16.6,
- $R$ LCL = 0

<table>
<thead>
<tr>
<th>Sample</th>
<th>Obs 1</th>
<th>Obs 2</th>
<th>Obs 3</th>
<th>Obs 4</th>
<th>$\bar{X}$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.8</td>
<td>32.9</td>
<td>29.3</td>
<td>31.9</td>
<td>30.7</td>
<td>4.1</td>
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<tr>
<td>2</td>
<td>29.4</td>
<td>27.9</td>
<td>26.8</td>
<td>32.8</td>
<td>29.2</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>27.7</td>
<td>28.8</td>
<td>26.3</td>
<td>32.1</td>
<td>28.7</td>
<td>5.8</td>
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<tr>
<td>4</td>
<td>32.9</td>
<td>22.9</td>
<td>24.5</td>
<td>35.3</td>
<td>28.9</td>
<td>12.4</td>
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<tr>
<td>5</td>
<td>36.4</td>
<td>32.2</td>
<td>28.3</td>
<td>30.4</td>
<td>31.8</td>
<td>8.1</td>
</tr>
</tbody>
</table>

5. [2 points] Based on the above data, is the process in control? Explain your answer for full credit.

Yes, each value of X-bar is within the control limits for the $\bar{X}$ chart and each value of R is within the control limits for the R chart.

Note: Most students got this correct.

The following description is for questions 6-7.

Libby’s manufacturing tracks a key dimension of a component in its tractors. Suppose that the mean of historical samples is 15.4 with a standard deviation of 4.1. Design specifications require that this quality dimension should be in the range from 10.6 to 20.6 to be non-defective.

6. [1 point] What is the $C_{pk}$?

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \min\left\{\frac{20.6 - 15.4}{3 \times 4.1}, \frac{15.4 - 10.6}{3 \times 4.1}\right\} = 0.390$$

Note: Most students got this correct.

7. [2 points] What is the probability of producing a non-defective unit?

USL = 20.6   LSL = 10.6
$\mu = 15.4, \sigma = 4.1$
Defective above USL: $Z > (20.6 - 15.4)/4.1 = 1.27$
This gives a probability of 0.898, so 1-0.898 = 0.102 above USL.
Defective below LSL: $Z < (10.6 - 15.4)/4.1 = -1.17$
This gives a probability of 0.121 below LSL.
Probability non-defective = 1 - 0.102 - 0.121 = 77.7%

Note: Most students got this correct.
The following description is for questions 8-10.
Park Sports is an outdoor retailer with two stores in upstate NY. The company is planning inventory for a particular type of rock climbing backpack. It buys this backpack from an overseas supplier for $60 each and sells it in its stores for $180 each. All leftover stock at the end of the season is sold to an outlet store for $45 each. Park Sports estimates that the estimated goodwill cost of being short, for each unit, is $5. The lead time to buy merchandise from the supplier is long, so the newsvendor model is applicable. Suppose that the demand for this backpack in each store is forecasted to have a normal distribution with mean 300 and standard deviation 65.

8. [2 points] How much inventory should be ordered, for each store, to maximize the expected profit?

\[ Cu = p - c + s = $180 - $60 + $5 = $125, \quad Co = c - v = $60 - $45 = $15 \]

Critical fractile = \( \frac{Cu}{Cu + Co} = \frac{125}{140} = 0.893 \)

\[ z = 1.25 \]

\[ Q = \text{mean} + z \times \sigma = 300 + 1.25 \times 65 = 381 \text{ (382 if roundup) backpacks for each store} \]

Note: Most students got this correct.

9. [1 point] One of the managers suggests that the company should consolidate the inventory for the backpacks in its nearby distribution center (DC). Under this plan, the company would only keep minimal merchandise in stores (assume zero) and quickly replenish stores from the DC when needed. Assume that the cost of shipping merchandise from the DC to stores is negligible, and the demand at the stores is unchanged and independent from one another. How much inventory should the company keep in the DC to maximize expected profit?

\[ z = 1.25 \text{ (the same as before), since the underage and overage costs are the same.} \]

\[ Q = 600 + 1.25 \times 91.92 = 714 \text{ (715 if roundup).} \]

Note: This result is due to risk pooling: standard deviation at each store is 65, so the pooled standard deviation = \( \sqrt{65^2 + 65^2} = 91.92 \)

Note: We did an example of this in class, the GAP example, where we discussed how one can add variances but not standard deviations.
The following description is for questions 8-10 (REPEATED).

Park Sports is an outdoor retailer with two stores in upstate NY. The company is planning inventory for a particular type of rock climbing backpack. It buys this backpack from an overseas supplier for $60 each and sells it in its stores for $180 each. All leftover stock at the end of the season is sold to an outlet store for $45 each. Park Sports estimates that the goodwill cost of being short, for each unit, is $5. The lead time to buy merchandise from the supplier is long, so the newsvendor model is applicable. Suppose that the demand for this backpack in each store is forecasted to have a normal distribution with mean 300 and standard deviation 65.

10. [2 points] Suppose Park Sports was unhappy with consolidating inventory at a single DC, and went back to the traditional model of holding inventory at each store. Further, Park Sports ultimately decided that it would be better to stock a number of backpacks equal to mean demand at each store: 300. Given this, what is their expected fill rate at each store?

For each store:
Q = 300
z = (300-300)/65 = 0
L(z) for z=0 is L(z=0) = 0.3989
E[Lost Sales] = sigma*L(z) = 65*0.3989 = 25.93 units
E[Sales] = D – E[LS] = 300 - 25.93 = 274.07 units

Fill Rate = E[Sales]/E[D] = 274.07/300 = 91.36%
Fill Rate = 1 - E[Lost Sales]/E[D] = 1 - 25.93/300 = 91.36%

Note: Most students got this correct.
The following description is for questions 11-14.

Waldman Soda makes three varieties of kombucha high-end cola: Regular, Diet, and Nil. The factory formulates the following linear program to determine how many cases of each type of cola to produce in the current season to achieve maximum contribution dollars (fractional cases are acceptable).

Decision Variables
- x: cases of Regular to produce
- y: cases of Diet to produce
- z: cases of Nil to produce

Linear Programming Formulation
Maximize $20x + 16y + 24z$ (dollars of contribution)
Subject to
- $4x + 2z \leq 3000$ (variety flavor A availability in pounds)
- $4y + 2z \leq 2040$ (variety flavor B availability in pounds)
- $x + 2z \leq 800$ (availability of sugar in lbs)
- $3x + y + 2z \leq 1000$ (man-hours of labor time available)
- $x, y, z \geq 0$

The sensitivity report obtained by running this program in Solver is below. Using this report, answer the questions that follow.

Sensitivity Analysis Report from Solver
Variable Cells

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Final Value</th>
<th>Reduced Cost</th>
<th>Objective Coefficient</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D8$</td>
<td>Cases Regular</td>
<td>0</td>
<td>-12</td>
<td>20</td>
<td>12</td>
<td>$1E+30$</td>
</tr>
<tr>
<td>$E8$</td>
<td>Cases Diet</td>
<td>346.7</td>
<td>0</td>
<td>16</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>$F8$</td>
<td>Cases Nil</td>
<td>326.7</td>
<td>0</td>
<td>24</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Constraints

<table>
<thead>
<tr>
<th>Cell</th>
<th>Name</th>
<th>Final Value</th>
<th>Shadow Price</th>
<th>Constraint R.H. Side</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G11$</td>
<td>Flavor A</td>
<td>653.3</td>
<td>0</td>
<td>3000</td>
<td>$1E+30$</td>
<td>2346.666667</td>
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<tr>
<td>$G12$</td>
<td>Flavor B</td>
<td>2040</td>
<td>1.3</td>
<td>2040</td>
<td>1960</td>
<td>440</td>
</tr>
<tr>
<td>$G13$</td>
<td>sugar</td>
<td>653.3</td>
<td>0</td>
<td>800</td>
<td>$1E+30$</td>
<td>146.666667</td>
</tr>
<tr>
<td>$G14$</td>
<td>man hrs</td>
<td>1000</td>
<td>10.7</td>
<td>1000</td>
<td>110</td>
<td>490</td>
</tr>
</tbody>
</table>
11. [1 point] What is the optimal objective function value (i.e. total contribution) in dollars?

\[ 0 \times 20 + 346.7 \times 16 + 326.7 \times 24 = \$13,388 \]

Note: Most students got this correct.

For questions 12-14, state whether each of the following changes (treat each independently) causes the optimal objective function value (i.e. total contribution) to increase, decrease, or stay the same, and by how much. Alternatively, if you believe an answer is not obtainable with the available information, then state that Solver must be re-run. Be as specific as possible.

12. [1 point] Suppose the number of labor man-hours decreases by 20 hrs, how does this affect the optimal objective function?

Decrease of $10.7 \times 20 = 214$, because shadow price is 10.7 and 20 is within the allowable decrease.

Note: Most students got this correct. For full credit, you needed to have the number, 214, and also specify that it will decrease.

13. [1 point] Suppose the price of Diet increases and its contribution goes up from $16 to $21 per case, how does this affect the optimal objective function?

Increase of roughly $5 \times 346.7 = \$1734$.

Note: Some students had trouble with this. The $5$ increase is within the allowable increase, so the same optimal solution remains, but the contribution increases by: \((21 - 16) \times 346.7 = \$1734\).

14. [1 point] Suppose Waldman Soda is able to acquire an extra 100 pounds of Sugar at $0 cost, how does this affect the optimal objective function?

No change, because the shadow price is zero and 100 is within the allowable increase.

Note: Most students got this correct.
15. [1 point] Consider the supply chain as shown below:

![Supply Chain Diagram]

Which of the following most closely represents the bullwhip effect in this supply chain (circle one)?

(a) \( \text{Var} (\text{Distributor’s Orders}) \geq \text{Var} (\text{Wholesale Orders}) \geq \text{Var} (\text{Retail Orders}) \geq \text{Var} (\text{Demand}) \)
(b) \( \text{Var} (\text{Distributor’s Orders}) \leq \text{Var} (\text{Wholesale Orders}) \leq \text{Var} (\text{Retail Orders}) \leq \text{Var} (\text{Demand}) \)
(c) \( \text{Var} (\text{Distributor’s Orders}) \leq \text{Var} (\text{Wholesale Orders}) \leq \text{Var} (\text{Retail Orders}) \geq \text{Var} (\text{Demand}) \)
(d) None of the above

16. [1 point] Suppose the critical fractile for a product, using the newsvendor model, is less than 0.5. What happens to the optimal quantity as the standard deviation of demand decreases, holding everything else constant (circle one)?

(a) \( Q^* \) increases
(b) \( Q^* \) decreases
(c) \( Q^* \) stays the same
(d) Cannot say without more information

17. [1 point] Toyota uses the Andon system to (circle one):

(a) level or balance the production
(b) make mistakes known
(c) reduce setup times
(d) achieve just-in-time production
(e) manage relationships with suppliers

18. [1 point] Suppose a company carries an inventory of undyed (i.e. no color) jacket, and allow customers to decide which color they want to purchase. At that time, the company would dye the jacket the appropriate color and ship it to the customer within one week. What does this fulfillment strategy most closely resemble (circle one)?

(a) Vendor Managed Inventory
(b) Delayed Differentiation
(c) Just in Time
(d) Toyota Production System
(e) Continuous Replenishment